Chapter 13 - Probability

Day 2 Sections 5 and 6

Lesson objectives: to learn the different types of probability; when to apply them to solve problems; and how to differentiate between them.

When playing Monopoly, is rolling the dice a dependent or independent event?

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take note

Key Concept **Probability of Mutually Exclusive Events**

If A and B are mutually exclusive events, then $P(A \text{ and } B) = 0$,

and $P(A \text{ or } B) = P(A) + P(B)$.

For example, the probability that you are in both Math and English is zero. The probability that you are in either Math or English is 1/7 + 1/7 = 2/7

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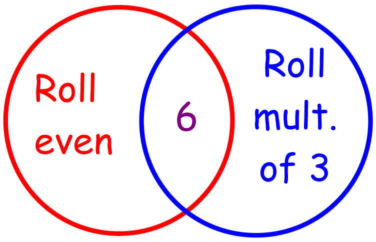
take note

Key Concept Probability of Overlapping Events

If A and B are overlapping events, then $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$.

Overlapping events have outcomes in common. For example, for a standard number cube, the event of rolling an even number and the event of rolling a multiple of 3 overlap because a roll of 6 is a favorable outcome for both events.



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$$P(\text{even or multiple of 3}) = P(\text{even}) + P(\text{multiple of 3}) - P(\text{even and multiple of 3})$$

$$= \frac{3}{6} + \frac{2}{6} - \frac{1}{6}$$

$$= \frac{4}{6}, \text{ or } \frac{2}{3}$$

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Two-way frequency tables

The table at the right shows the number of students who passed their driving test as well as whether they took a driver's education class to prepare. What effect, if any, does taking the driver's education class have?

	Passed	Failed	Totals
Took the class	32	7	39
Do not take the class	18	23	41
Totals	50	30	80

A two-way frequency table, or contingency table, displays the frequencies of data in two different categories.

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Activities The table shows data about student involvement in extracurricular activities at a local high school. What is the probability that a randomly chosen student is a female who is not involved in extracurricular activities?

Extracurricular Activities

	Involved in Activities	Not Involved in Activities	Totals
Male	112	145	257
Female	139	120	259
Totals	251	265	516

To find the probability, calculate the relative frequency.

$$\text{relative frequency} = \frac{\text{females not involved}}{\text{total number of students}} = \frac{120}{516} \approx 0.233$$

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Extracurricular Activities

	Involved in Activities	Not Involved in Activities	Totals
Male	112	145	257
Female	139	120	259
Totals	251	265	516

How many students are male?
How many students are female?
How many students are involved in activities?
What is the probability that a random male student is involved in activities?

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The two-way frequency table at the right shows the number of male and female students by grade level on the prom committee. What is the probability that a member of the prom committee is a male who is a junior?

	Male	Female	Totals
Juniors	3	4	7
Seniors	3	2	5
Totals	6	6	12

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The probability that an event will occur, given that another event has already occurred is called a **conditional probability**. You can write the conditional probability of event B, given that event A has already occurred as $P(B|A)$. You read $P(B|A)$ as "the probability of event B, given event A."

Age Group	For	Against	No Opinion	Totals
18–29	310	50	20	380
30–45	200	30	10	240
45–60	120	20	30	170
Over 60	150	20	40	210
Totals	780	120	100	1000

Find P(over 60 | no opinion)

$$P(\text{over } 60 \mid \text{no opinion}) = \frac{40}{100} = 0.4$$

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Business A company has 150 sales representatives. Two months after a sales seminar, the company vice-president made the table of relative frequencies based on sales results. What is the probability that someone who attended the seminar had an increase in sales?

	Attended Seminar	Did not Attend Seminar	Totals
Increased Sales	0.48	0.02	0.5
No Increase in Sales	0.32	0.18	0.5
Totals	0.8	0.2	1

Method 1

Find frequencies first.

Find the number of people who attended the seminar and had increased sales: $0.48 \cdot 150 = 72$

Find the number of people who attended the seminar: $0.8 \cdot 150 = 120$

Find $P(\text{increased sales} \mid \text{sales seminar})$: $\frac{72}{120} = 0.6$, or 60%

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	Attended Seminar	Did not Attend Seminar	Totals
Increased Sales	0.48	0.02	0.5
No Increase in Sales	0.32	0.18	0.5
Totals	0.8	0.2	1

Method 2

Use relative frequencies.

$P(\text{increased sales} \mid \text{sales seminar})$:

$$= \frac{\text{relative frequency of attend seminar and increased sales}}{\text{relative frequency of attended seminar}} = \frac{0.48}{0.8} = 0.6$$

Q: What is the probability that a randomly selected sales representative, who did not attend the seminar, did not see an increase in sales?

0.9 or 90%

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Conditional Probability Formulas

	Attended Seminar	Did not Attend Seminar	Totals
Increased Sales	0.48	0.02	0.5
No Increase in Sales	0.32	0.18	0.5
Totals	0.8	0.2	1

$$P(\text{increased sales} \mid \text{attend seminar}) = \frac{P(\text{attend seminar and had increased sales})}{P(\text{attended seminar})}.$$

For any two events A and B, the probability of B occurring, given that event A has occurred, is

$$P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)}, \text{ where } P(A) \neq 0.$$

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Pharmaceutical Testing In a study designed to test the effectiveness of a new drug, half of the volunteers received the drug. The other half of the volunteers received a placebo, a tablet or pill containing no medication. The probability of a volunteer receiving the drug and getting well was 45%. What is the probability of someone getting well, given that he receives the drug?

Step 1 Identify the probabilities.

$P(B|A) = P(\text{getting well, given taking the new drug})$

$P(A) = P(\text{taking the new drug}) = \frac{1}{2} = 0.5$

$P(A \text{ and } B) = P(\text{taking the new drug and getting well}) = 45\%$, or 0.45

Step 2 Find $P(B|A)$.

$P(B|A) = \frac{0.45}{0.5} = 0.9$, or 90% Use the conditional probability formula.

Q: The probability of a volunteer receiving the placebo and having his or her health improve was 20%. What is the conditional probability of a volunteer's health improving, given that they received the placebo? 0.4 or 40%

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Pets In a survey of pet owners, 45% own a dog, 27% own a cat, and 12% own both a dog and a cat. What is the conditional probability that a dog owner also owns a cat? What is the conditional probability that a cat owner also owns a dog?

$P(\text{cat}\mid\text{dog}) = \dfrac{P(\text{owns cat and dog})}{P(\text{owns dog})}$	Definition	$P(\text{dog}\mid\text{cat}) = \dfrac{P(\text{owns cat and dog})}{P(\text{owns cat})}$
$= \dfrac{0.12}{0.45}$	Substitute.	$= \dfrac{0.12}{0.27}$
≈ 0.267, or 26.7%	Simplify.	≈ 0.444, or 44.4%

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Isolating P(A and B)

Because $P(B \mid A) = \dfrac{P(A \text{ and } B)}{P(A)}$, then $P(A \text{ and } B) = P(A) \cdot P(B \mid A)$.

You can use this form of the conditional rule when you know the conditional probability. You can also combine conditional probabilities to find the probability of an event that can happen in more than one way.

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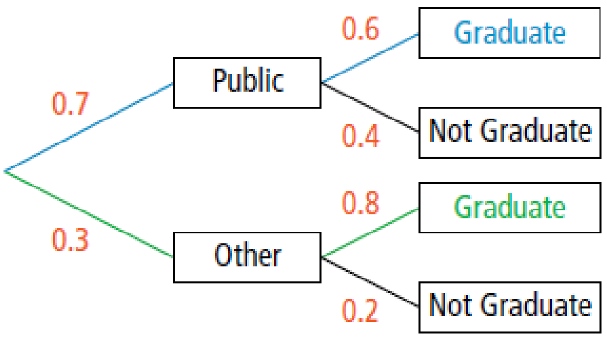
Using a Tree Diagram

Graduation Rate A college reported the following based on their graduation data.

- 70% of freshmen had attended public schools
- 60% of freshmen who had attended public schools graduated within 5 years
- 80% of other freshmen graduated within 5 years

What percent of freshmen graduated within 5 years?



$= 0.6 \cdot 0.7$
$= 0.42$

$= 0.8 \cdot 0.3$
$= 0.24$

$$\boxed{\begin{aligned} &= 0.42 + 0.24 \\ &= 0.66 \end{aligned}}$$

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